Blair's Problems *du jour*

23rd January 2007

1 M-estimation

Sorry for the bother, but I wondered if the solution to this problem was trivial, with the usual $A \in R^{n\times m}$, where $n \gg m$ most of the time, and $b \in R^n$.

$$\min \|Ax-b\|_1$$
$$\|x\|_j \leq \tau$$

for $j = 1, 2$. And I define $\|b\|_i = \sum_{j=1}^n |b|^i$. Obviously, everything is well know when using least squares. I'm looking for a) methods to solve this and b) anything interesting in the dual or its solution. I think the general name for this would be something like penalized M-estimation in statistics (as opposed to constrained M-estimation which is something a little different).

2 Fast Solutions of Many Related Least Squares Solutions

In this setup, I have the following 2 part problem. First, the setup changes a little here. At this point, I can say that A is sparse, specifically, I can say that at least part of A that the columns (A_i) have a special property, which I can reparameterize to maximize/minimize the sparseness of A. Specifically, I start my parameterization such that I have columns of ones and zeros who add to the vector of ones. When creating A, for statistical reasons, we define the first column of A to be the vector of ones. Then we drop one of the columns from before so that A will be full rank. Thus A is between $\frac{1}{3}$ and $\frac{2}{3}$ sparse in this case. (by sparse, I mean that percent of the entries are zero, the others are ones). I actually have this problem in different contexts. The structure is the same for all A_i in a given context. The other contexts are some variations on this with columns of A which have reals or other 0/1 columns.

For the first part, I am solving $\min \|A_i x - b\|_2$. And $i = 1, \cdots, n$, where, right now, $n \sim 500k$, and next year $n \sim 1.5e7$. As $n \to \infty$, $Cor\left(\min \|A_i x - b\|_2, \min \|A_{i+1} x - b\|_2\right) \to 1$. Pardon the total abuse of notation, but I wanted to convey the idea that, for the most part, (there are gaps), the solutions to $\min \|A_i x - b\|_2$ and $\min \|A_{i+1} x - b\|_2$ are very close, and tend to get closer as n increases. The second part goes like this: I also need to solve $(A'A)^{-1}$ quickly, and I'm not sure what the best way to approach that is.

I spoke with our post-doc Chris Calderon about this, and he mentioned that something iterative like conjugate residual would be appropriate here. There's also another twist. I have been solving this by taking $A = QR$, but I also need the variance estimate, which I currently get from $k\,(A'A)^{-1}$, and currently, this is nice, $k\,(R'Q'QR)^{-1} = k\,(R'R)^{-1}$ which inverts fine using the algorithm for inverting a cholesky decomposition. I'm not sure how easy it would be to also get this $(A'A)^{-1}$ quickly? Is there another iterative algorithm I could use here? Or is there a trick to get it to fall out of